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Spring 2023 Math 481 Mathematical Theory of Statistics

L3 Worksheet. MGF, sample mean (8.1, 8.2)

The Gamma distribution is specified by two parameters positive α, β. Its MGF is $M(t) = \frac{1}{(1-\beta t)^{\alpha}}$.

1. Suppose X is a random variable with Gamma distribution with $\alpha = 3, \beta = 2$. Use the MGF to compute expectation $E[X]$ and variance $Var[X]$.

$(1-\beta t)^{-\alpha}$ $\quad \alpha\beta(1-\beta t)^{-\alpha-1}$

$E[X] = 6 \cdot 1^{-4} = 6$

$\alpha(\alpha-1)\beta(-\beta)(1-\beta t)^{-\alpha-2}$

$3 \quad -4 \quad 2 \quad -2 \quad 1^{-5}$

$E[X^2] = 48$

$Var[X] = 48 - 6^2 = 12$

2.

3. Compute the MGF of sample mean $\overline{X}$ of a random sample of size n from a Gamma population with $\alpha = 3, \beta = 2$. Is $\overline{X}$ a Gamma random variable? Why or why not.

$\overline{X} = \frac{X_1 + X_2 + \ldots X_n}{n}$

$M_{\bar{X}}(t) = M_X(t/n)^n = (1-2t/n)^{-3n} = \frac{1}{(1-2/n\, t)^{3 \cdot n}}$

$\bar{X}$ is a gamma random variable with $\alpha = 3n$ and $\beta = 2/n$

4. Continuing previous question: What is the mean and variance of $\overline{X}$

$3n \cdot 2/n \cdot 1^{-3n-1} = 6$ $\quad E[\bar{X}] = E[X] = 6$

$3n(-3n-1)2/n(-2/n)1^{-3n-2} = 6(-3n-1)(-2/n) = 12/n(3n+1) = 36 + 12/n = E[\bar{x}^2]$

$Var[\bar{X}] = E[\bar{x}^2] - E[\bar{x}]^2 = 36 + 12/n - 36 = 12/n$